The Future of Magnets

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Abstract. Even though magnetism belongs to the oldest known phenomena in physics the topic continues to play a significant role in both research and technology. With regards to magnetic materials ferromagnetic insulators are of particular importance due to their applications in spintronics and their ability to inject spin currents. Ferromagnetic insulators are theoretically described using the sf-model. From its exactly solvable limiting cases important results with regards to the density of states are obtained and discussed. Magnets as energy storage systems, advanced MRI techniques using superconducting magnets, and superconducting spin currents are presented as modern applications to magnetic systems. Furthermore, spin flip scattering and magnon absorption at the interface between a ferromagnetic insulator and a superconductor give rise to a phase transition from a superconducting to an ordinary metallic phase. It is shown that this phase transition is of second order.

Keywords: sf-model, magnon emission, spin currents, ferromagnetic insulator superconductor interface.

1.Introduction With regards to magnetic materials ferromagnetic insulators are of particular importance due to their applications in spintronics. In spintronics or spin transport electronics the results of conventional magnetism and semiconductor physics are correlated. Storage and processing of data is brought together on a single chip utilizing both the electron's charge and its spin. The aim is to make appliances smaller and smaller while at the same time optimizing efficiency and reducing costs. To achieve this spin dependent electron transport phenomena are desirable [1,2] and in this respect crystals with atoms from the group of Rare Earth elements become important. These atoms contain partially filled 4f-shells that are localized at the lattice site and are responsible for the magnetic moment of the atom, while the 6s-electrons become the quasi-free conduction electrons that can move through the entire system. Magnetism and the electric current are thus caused by two different electron groups and it is precisely the interaction between these electron groups that is responsible for the many interesting phenomena in these materials, for example RKKY interaction, Kondo and spinglass behavior.

It is the aim of this article to explain the injection of spin currents at the ferromagnetic insulator/ superconductor interface as reported in the experimental study of reference]3]. As magnetic materials gadolinium gallium garnets are used to grow superconducting films. It was concluded that spin flip scattering and magnon absorption are responsible for spin dependent electron transport phenomena and conduction electron spin polarizations $P\ (T,n) \neq 0$ in these materials.

It is well documented in the literature that the sf-model describes magnetism in magnetic semiconductors reasonably well [4,5]. The corresponding Hamiltonian is presented in Section 2 of this article. Model calculations of this type are less accurate than first principles methods based on DFT calculations and often only allow a qualitative comparison with the experiment. However, model calculations have the advantage that phenomena like magnetism can be explained from specific interaction terms in the Hamiltonian; similar conclusions cannot be drawn with the same conviction from DFT calculations. W Nolting et al [6,7] combine the many body problem of the sf-model with a selfconsistent band structure calculation based on DFT to obtain highly realistic results for the Curie temperature T_C of ferromagnetic 4f-systems.

The many body problem defined by the sf-model is generally not exactly solvable. An interesting exactly solvable limit, namely the special case ($T = 0, n = 0$) [8] is presented in Section 3. One important result of this special case are spin flip scattering and magnon absorption which are then used to explain spin transport phenomena at the ferromagnetic insulator/superconductor interface. A numerical evaluation indicates a possible coexistence between superconductivity and ferromagnetic order as defined by the Stoner criterion of reference [9]. Finally, in Section 5 applications to energy storage systems and magnets in medicine are discussed.

2. The Model The sf-model describes the local interaction between the two different electron groups in magnetic semiconductors and is defined by the Hamiltonian

$$H = H_s + H_f + H_{sf}$$

$$H_s = \sum_{ij\sigma} T_{ij}\, c_{i\sigma}^{+}\, c_{j\sigma} + \frac{1}{2}\, U \sum_{i\sigma} n_{i\sigma}\, n_{i-\sigma} \tag{1}$$

Here $c_{i\sigma}^{+}$ denotes the creation operator for a σ-electron at lattice site R_i, $c_{i\sigma}$ is the corresponding annihilation operator and

$$n_{i\sigma} = c_{i\sigma}^{+}\, c_{i\sigma}$$

is the operator of the occupation number. U is the intraatomic Coulomb matrix element while the T_{ij} represent the hopping integrals. They are related to the Bloch energies via Fourier transformation, namely

$$T_{ij} = \frac{1}{N} \sum_{k} \varepsilon\,(k)\;\; e^{ik\,\cdot\,(R_i - R_j)}$$

H_s describes the system of itinerant conduction electrons that are treated as s-electrons and has the well known form of the Hubbard model. The subsystem of localized magnetic moments is described in a realistic manner by the Heisenberg model

$$H_f = -\sum_{ij} J_{ij}\, S_i \cdot S_j \tag{2}$$

The spins at R_i and R_j interact via the exchange integrals J_{ij} that are conveniently restricted to nearest neighbors. The two subsystems are coupled by an sf-exchange, i.e. a local interaction between the 4f-spin S_i and the conduction electron spin σ_i

$$H_{sf} = -g \sum_i \sigma_i \cdot S_i = -\frac{1}{2}\, g \sum_{i\sigma} (z_\sigma\, S_i^z\, n_{i\sigma} + S_i^\sigma\, c_{i-\sigma}^{+}\, c_{i\sigma}) \tag{3}$$

g is the intraatomic sf-exchange constant. Note that the second term of Eq (3) describes spinflip processes where an electron at lattice site R_i and spin σ is annihilated and a corresponding electron of opposite spin is created; the first term is a diagonal contribution containing the z-component of the spin operator.

The Hamiltonian of Eq (1) describes a non-trivial many body problem that is generally not exactly solvable. However, there are a couple of exactly solvable limiting cases available, namely the $T = 0\,K$ case of one electron in an otherwise empty conduction band and the zero bandwidth limit. The former case explains the existence of the magnetic polaron [10] and is discussed in the following section.

Exactly solvable limiting cases are important to test the accuracy of standard approximative procedures by numerically determining the discrepancy between the exact and the approximate result and from that draw conclusions regarding the more general many body problem.

2.1 The special case $T = 0, n = 0$

This interesting and exactly solvable special case describes the situation of one electron in an otherwise empty conduction band. One distinguishes between the two spin states $\sigma = \uparrow\downarrow$. For the case $\sigma = \uparrow$ we obtain for the one electron Green function and density of states respectively

$$G_{k\uparrow}\,(E) = \frac{\hbar}{E - \varepsilon\,(k) + \frac{1}{2} g\, S}$$

$$\rho_\uparrow\,(E) = \rho_0\,(E + \frac{1}{2}\, gS\,) \tag{4}$$

The density of states is rigidly shifted with respect to the free Bloch band. On the other hand, the case $\sigma = \downarrow$ turns out to be slightly more complicated as

$$G_{k\downarrow}\,(E) = \frac{\hbar}{E - \varepsilon\,(k) - M_\downarrow\,(E)} \tag{5}$$

Here $M_{\downarrow}(E)$ denotes the complex selfenergy representing real many body effects. As a consequence there are now two subbands, the lower of which describes spin flip scattering whereby a magnon of energy $\hbar\ \omega\ (q)$ is emitted. This scattering spectrum lies in the same energy region as the $\sigma = \uparrow$ density of states. On the other hand, the upper band represented by a δ- function in the spectral density $S_{k\downarrow}(E)$ describes a bound state formed with an antiparallel 4f-spin. The corresponding quasiparticle is the above mentioned magnetic polaron.

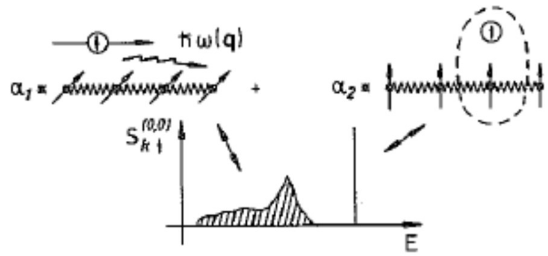

Fig 1: Spectral density $S_{k\downarrow}(E)$ for the case $T = 0, n = 0$, adopted from reference [10].

An interesting quantity that can be straightforwardly calculated for the special case $T = 0$ is the conduction electron spin polarization

$$P\ (T, n) = \frac{n_{\uparrow} - n_{\downarrow}}{n_{\uparrow} + n_{\downarrow}} \quad (6)$$

It is plotted in Figure 2 below as a function of band occupation n.

The system of conduction electrons is for practically all band occupations n at least to some degree polarized which again proves the applicability of ferromagnetic semiconductors as candidates to produce spin currents.. The only exception is the case $n = 2$ which represents a fully occupied conduction band. For this case we obtain $n_{\uparrow} = n_{\downarrow}$.

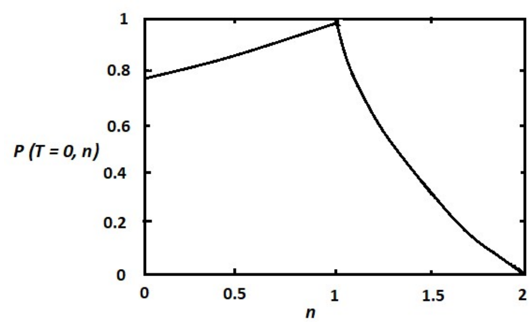

Figure 2 Conduction electron spin polarization $P(T = 0, n)$ as a function of band occupation n.

In the next section we will discuss the consequences of these results with regards to ferromagnetic insulator/ superconductor interfaces.

3. Results The authors of the experimental study of reference [3] report spin currents being injected at the ferromagnetic insulator/superconductor interface. As the ferromagnetic material gadolinium gallium garnets can be used as they are capable of growing superconducting films and conducting spin transport experiments. They are furthermore characterized by a small exchange coupling parameter J resulting in an equally low Curie temperature T_C, the absence of long range magnetic order and a statistical distribution of magnetic ions leading to spin glass behavior and magnetic frustration [11].

On the other hand, superconductivity is generally believed to be caused by a phonon induced attractive contribution to the electron-electron interaction leading to the formation of Cooper pairs at low temperatures. This view is confirmed in the work by L Fritsche [12] where it is argued that for the superconducting state to have a lower total energy than the metallic state the theory must go beyond the Born-Oppenheimer approximation thus making lattice vibrations indispensable for the understanding of the phenomenon.

We want to investigate under what kind of conditions superconductivity becomes energetically favorable and for that purpose evaluate the Cooper pair energy

$$E = 2\,\varepsilon_F - 2\,\hbar\,\omega_D\,\frac{e^{-1/x}}{1-e^{-1/x}} \qquad (7)$$

as a function of the parameter $x = U\,\rho_0\,(\,\varepsilon_F\,)$. Here U denotes the intraatomic Coulomb interaction while $\rho_0\,(\,\varepsilon_F\,)$ represents the density of states at the Fermi energy. Corresponding results are plotted in Fig 3 below.

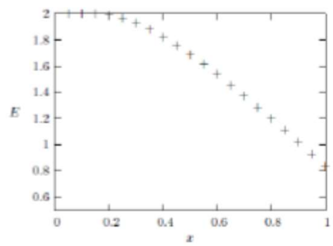

Fig 3: Cooper pair energy of Eq (7) above as a function of the parameter x.

At small values of x

$$E \cong 2\,\varepsilon_F \qquad (8)$$

which is equal to the energy of two non-interacting electrons. With increasing parameter x the energy continuously decreases making the superconducting phase energetically more favorable. This suggests a possible coexistence between superconductivity and ferromagnetic order as suggested by the Stoner criterion

$$U\,\rho_0\,(\,\varepsilon_F\,) > 1 \qquad (9)$$

for ferromagnetism [9]. Superconductivity and ferromagnetic order are thus energetically favorable in the same region of x-values and this phenomenon has also been reported by various other authors [13,14,15].
Interesting in this context is also the superconducting gap parameter $\Delta\,(T)$ as a function of temperature T. It is the energy gap between the BCS ground state and the first excited state and should thus vanish at the critical temperature T_C of superconductivity. This critical temperature is a material property, is usually of the order of a few Kelvin, but in high temperature superconductors can become close to room temperature.

Corresponding results are plotted in Fig 4 below.

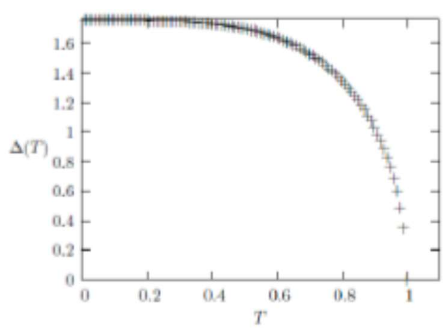

Fig 4: Superconducting gap parameter $\Delta\,(T)$ as a function of the reduced temperature T/T_C.

At $T \rightarrow 0$ the results of the BCS theory [16] are reproduced. With increasing temperature the gap parameter Δ continuously decreases until finally vanishing at $T \rightarrow T_C$. At $T > T_C$ the normal metallic phase is observed. The phase transition is of 2nd order with an order parameter $\beta = \frac{1}{2}$.
The functional behavior of Fig 4 also qualitatively agrees with the results of reference]3].

4. Conclusions

Ferromagnetic insulators are important to both research and technology due to their applications in spintronics. They are theoretically described by the sf-model; its $T = 0$ special case gives rise to spin flip scattering and magnon emission processes.
These results were then used to explain the injection of spin currents at the ferromagnetic insulator/superconductor interface. The evaluation of the Cooper pair energy indicates a possible coexistence between superconductivity and ferromagnetic order. The superconducting gap parameter $\Delta\ (T)$ is evaluated as a function of temperature T and a 2nd order phase transition is observed with an order parameter $\beta = {}^{1}/_{2}$.
Important applications of magnets as energy storage systems (MRAM and SMES devices) and magnets in medicine (for targeted cancer treatment [17[) plus the discovery of new magnetic materials with stronger magnetic fields are responsible for the fact that magnetic materials continue to play a significant role in both research and technology.

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